

# Refined Applications of the “Collapse of the Wavefunction”

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## Abstract

In a two-part system the “collapse of the wavefunction” of one part can put the other part in a state which would be difficult or impossible to achieve otherwise, in particular one sensitive to small effects in the ‘collapse’ interaction.

We present some applications to the very symmetric and experimentally accessible situations of the decays  $\phi(1020) \rightarrow K^0 K^0$ ,  $\psi(3770) \rightarrow D^0 D^0$ , or  $\Upsilon(4s) \rightarrow B^0 B^0$ , involving the internal state of the two-state  $K^0 D^0$  or  $B^0$  mesons. The “collapse of the wavefunction” occasioned by a decay of one member of the pair (‘away side’) fixes the state vector of that side’s two-state system. Bose-Einstein statistics then determines the state of the recoiling meson (‘near side’), whose evolution can then be followed further.

In particular the statistics requirement dictates that the ‘away side’ and ‘near side’ internal wavefunctions must be orthogonal at the time of the “collapse”. Thus a CP violation in the ‘away side’ decay implies a complementary CP impurity on the ‘near side’, which can be detected in the further evolution. The CP violation so manifested is necessarily *direct* CP violation, since neither the mass matrix nor time evolution was involved in the “collapse”.

A parametrization of the direct CP violation is given and various manifestations are presented. Certain rates or combination of rates are identified which are nonzero only if there is direct CP violation.

The very explicit and detailed use made of “collapse of the wavefunction” makes the procedure interesting with respect to the fundamentals of quantum mechanics. We note an experimental consistency test for our treatment of the “collapse of the wavefunction”, which can be carried out by a certain measurement of partial decay rates.

## 1 Introduction

The “collapse of the wavefunction”, where a “measurement” suddenly fixes the state of a quantum mechanical system, is one of the longest discussed and most

difficult chapters in quantum mechanics. This is especially true when the ‘collapse’ is used to produce the EPR ‘paradox’ [1], where the ‘measurement’ on one part of a system fixes the state of another, remote, part of the same system.

While the physical and philosophical discussion continues almost unabated, over the years there have been what might be called ‘practical’ applications of the “collapse” and the “paradox”. In 1968 Lipkin [2] proposed using it to study properties of  $K^0$  decays, and in  $B^0$  physics [3], where one studies the decay  $\Upsilon(4s) \rightarrow B^0 B^0$ , it has been used to study CP violation [4]. One uses the decay of one member of the pair into a given flavor state to determine that the other member is in the opposite flavor state, at the same time. Furthermore, we recently explained [5] how the “collapse” can be used to circumvent spatial resolution difficulties connected with the relatively large size of the beam crossing region at the  $e^+e^-$  colliders where the  $\Upsilon(4s)$  is produced.

While in the applications to states of definite flavor one might dismiss the results as consequences of simple flavor conservation in strong interactions, one also has, as explained for example in ref [5], applications to other, more non-trivial, states of the two-state system. In this paper we would like to study such further, more subtle application of the “collapse of the wavefunction”, and to present a systematic formalism allowing a general treatment of the  $K^0 D^0$  or  $B^0$  decays in the systems  $\phi(1020) \rightarrow K^0 K^0$ ,  $\psi(3770) \rightarrow D^0 D^0$ , or  $\Upsilon(4s) \rightarrow B^0 B^0$ . In particular we will note applications to CP violation— particularly *direct* CP violation.

The concept of the “collapse of the wavefunction” undoubtedly brings a number of conceptual and psychological difficulties with it, and we believe these can be lifted by using the amplitude and not the wavefunction as the fundamental quantity [6]. However, for the present purposes it appears convenient and more familiar to work in the wavefunction approach, and in the following we will take the ‘collapse’ quite literally. In section 16.1 we mention an experimental test of our interpretation of the ‘collapse’. Finally, it is possible that the principle can be applied in other fields of physics as well, but we shall not go into this here.

## 2 The Principle

Briefly, our idea is that the “collapse of the wavefunction” of one part of a systems can be used to put the other part in a quantum mechanical state which reflects features of the interactions involved in the ‘collapse’. In this way small effects, like CP violation in the ‘collapse’ amplitudes, can be put into direct evidence.

Let a system consist of two coherent parts, as in the decay  $\phi(1020) \rightarrow K^0 K^0$ . A ‘measurement’ on one part (we will call this the ‘away side’) fixes –‘collapses’– the wavefunction of that part. Often the other part (called the ‘near side’)– will be connected to the first part by some symmetry or conservation principle which correlates<sup>1</sup> the wavefunctions of the two parts. The ‘collapse’ on the ‘away side’ thus determines or partially determines the wavefunction of the ‘near side’.

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<sup>1</sup>In quanto-babble these correlations are often called ‘entanglement’.

Hence by observing the further behavior of the ‘near, uncollapsed, side’ one can obtain information on the interactions inducing the ‘collapse’. We shall pursue the idea that this information can include even small effects, like those due to direct CP violation.

### 3 The $\phi(1020)$ , $\psi(3770)$ , $\Upsilon(4s)$ systems

Because there is a high degree of symmetry and simplicity in the p-wave decays of the  $\phi(1020)$ ,  $\psi(3770)$ , and  $\Upsilon(4s)$  to a particle anti-particle pair, where each is a two-state system, namely  $K^0, D^0$  or  $B^0$  respectively, these are interesting and experimentally accessible cases. In addition to the interest of the quantum mechanical principles, it can also offer a way to determine some of the parameters of the  $K^0, D^0$  or  $B^0$  systems. Indeed, the first suggestion of this general kind, by Lipkin [2] concerned CP violation in  $K^0$  decay.

The  $K^0, D^0$  or  $B^0$  we deal with is a two-state system, represented by a two-dimensional linear vector space. This can be conveniently visualized as a kind of ‘spin 1/2’, an analogy we shall use in the following. These particles will in turn decay into certain final states, the decay “channels”. We shall refer to the first particle to decay as the ‘away side’, and the undecayed one, the one we may follow further, the ‘near side’.

We will utilize two major points concerning the two-state  $K^0, D^0$  or  $B^0$  :

1) The decay into a specific channel, like  $\pi^+\pi^-$  or  $\pi^0\pi^0$  for the  $K^0$ , is tantamount to a ‘measurement’ and fixes the state of the originating system (the  $K^0$ ) in its two-dimensional space; the ‘spin’ is fixed in some definite ‘direction’. Of course, all wavefunctions can have an arbitrary overall phase factor.

2) Bose-Einstein statistics in the form of an overall symmetry of the wavefunction applies to the identical meson pair. For the  $\phi(1020), \psi(3770)$ , or  $\Upsilon(4s)$  one has  $l = 1$  decays, so that the spatial wavefunction is antisymmetric. Therefore the internal wavefunction of the identical mesons must also be antisymmetric. This in turn implies that the two-state vectors of the mesons are orthogonal. In section 15 we briefly examine the opposite case, where the internal wavefunction is symmetric.

Another way of expressing 2) is to say that given a certain channel on one side, the other side (at the *same time*) must be precisely that state which *cannot* decay into the given channel. Otherwise there would be a violation of Bose-Einstein statistics—identical systems in a p-wave [2]. Note no further symmetries like CP are involved in this statement.

This statement 2) describes the correlation mentioned above which serves to fix the ‘near side’ once the ‘away side’ has been ‘collapsed’. In other applications the type of correlation can of course be different, as with an s-wave pair (section 15).

## 4 Eigenstate for a Decay

In appreciating the first statement 1) it is important to recognize, as has been stressed in refs.[5], [7], that a decay channel, call it  $a$ , defines a certain, unique, state of the two-state  $K^o$   $D^o$  or  $B^o$ . For a two-state system there are two amplitudes for the decay to channel  $a$ . In, say, the flavor basis for the  $K^o$  system, there is one amplitude for the  $K^o$  decay and one for the  $\bar{K}^o$  decay, call these amplitudes  $\alpha$  and  $\alpha'$ . Now with one channel and two states, it is always possible to find a state which does *not* decay into the given channel: namely the state  $\alpha' |K^o\rangle - \alpha |\bar{K}^o\rangle$ . This state has the decay amplitude  $\sim \alpha\alpha' - \alpha'\alpha = 0$  and evidently does *not* go into the channel  $a$ . On the other hand, the state orthogonal to this no-decay state, namely  $\alpha^* |K^o\rangle + \alpha'^* |\bar{K}^o\rangle$ , has the decay amplitude  $\sim |\alpha|^2 + |\alpha'|^2$  and *does* decay into the channel  $a$ . Hence given a sample of events with ‘away side’  $a$ , the state vector of the ‘away side’ is uniquely determined (up to the overall phase factor). We shall assume that the fact that a decay has occurred means the system was in that state, with no component of the other, not-allowed-to-decay, state. In section 16.1 we propose a test of this assumption.

At the same time, this also fixes the orthogonal state or ‘particle’, of the two-state system on the ‘near side’; it is the one which does *not* go into the channel  $a$ .

## 5 Direct CP violation

Of course in the presence of some exactly conserved quantum number such as CP, channels like  $K^o \rightarrow \pi^+\pi^-$  and  $K^o \rightarrow \pi^o\pi^o$  can define the *same* state of the parent, a state of definite CP, the CP=+1 state called  $|K_1\rangle = \frac{1}{\sqrt{2}}(|K^o\rangle + |\bar{K}^o\rangle)$ , and thus determining the orthogonal state to be  $|K_2\rangle = \frac{1}{\sqrt{2}}(|K^o\rangle - |\bar{K}^o\rangle)$  on the ‘near side’.

However, if CP is not exactly conserved, we may expect that the ‘away side’ state vector so determined is not precisely a state of pure CP, nor is it necessarily the same for different channels like  $\pi^+\pi^-$  or  $\pi^o\pi^o$ . It is these possible small differences we would like to examine for the study of direct CP violation.

One may thus anticipate a number of experimental consequences of direct CP violation on the ‘away side’ which can be studied in the behavior of the ‘near side’. These will be discussed systematically below, but we mention two simple ones which come quickly to mind

**Two-Channel Difference**– Consider two different ‘away side’ channels, of the same CP like  $\pi^+\pi^-$  and  $\pi^o\pi^o$  for the  $K^o$ . If CP were perfectly conserved in the respective ‘collapses’, then both channels would define exactly the same ‘near side’ state at  $t = 0$ , namely  $K_2$ . But with direct CP violation in the ‘collapse’, the ‘near side’ may be different for the two channels. Hence *any* differences at all in the evolution of the ‘near side’ for the two data samples  $\pi^+\pi^-$  or  $\pi^o\pi^o$  implies there was a CP violation on the ‘away side’. (section 13).

**Manifest CP impurities**– Decays on the ‘near side’ can show a manifest

particle –antiparticle asymmetry when the ‘away side’ amplitudes have direct CP violation. This is particularly simple at  $t = 0$ , where  $t = 0$  is the time of the ‘collapse’ of the ‘away side’(Eq 14).

We stress that these CP violations necessarily represent *direct* CP violation, since only direct decay amplitudes, with no time evolution and thus effects of the mass matrix, are involved.

## 6 Paramaterization

To proceed, we define a parameter  $\zeta$  characterizing the CP violation in the ‘away side’ decay amplitude. Continuing to use the  $K^o$  system to exemplify the ideas, we use the basis of CP eigenstates  $K_1, K_2$  and consider decays to states of definite CP. By “states of definite CP” we mean those states, like  $\pi^+\pi^-$  or  $(J/\psi)K_s$  which would be CP eigenstates in the CP conserving limit. For explicitness we shall usually refer to  $K_1, K_2$ , but the discussion applies to  $D^o$  or  $B^o$  equally well, using  $D_1, D_2$  or  $B_1, B_2$  states.

The direct CP violation will be manifested as an amplitude from the ‘wrong’ CP state into the decay channel in question. We thus define

$$\zeta = \frac{\alpha'^*}{\alpha^*} = \frac{(CP\text{ violating amplitude})^*}{(CP\text{ conserving amplitude})^*} \quad (1)$$

With a CP=+1 decay channel the numerator is thus the complex conjugate of the  $K_2$  decay amplitude and the denominator is the complex conjugate of the  $K_1$  amplitude. With a CP=-1 decay channel the identifications are reversed. The connection of this characterization of direct CP violation with the traditional notation is discussed in the Appendix.

### 6.1 States

Thus the normalized states which can decay into a channel characterized by  $\zeta$  are for CP=+1 channels,  $|K_\zeta\rangle = \frac{1}{\sqrt{1+|\zeta|^2}}(|K_1\rangle + \zeta|K_2\rangle)$ , and for CP=-1,  $|K_\zeta\rangle = \frac{1}{\sqrt{1+|\zeta|^2}}(\zeta|K_1\rangle + |K_2\rangle)$ . While in the  $\zeta = 0$  limit these states are of course orthogonal, in general there is no particular statement about the orthogonality or not of these two states. In any case the  $\zeta$ ’s depend on the decay channel in question.

In Table 1 we show these states, together with their orthogonal states. Since we will sometimes need the states in the flavor basis we show this also. It will be seen that the orthogonal states  $|K_{\zeta\perp}\rangle$  are just those that do not decay into the given channel.

### 6.2 Probabilities

To consider the further development of the ‘near side’ after the ‘collapse’, the evolution of the states is most conveniently expressed in a density matrix formalism, as was used in ref [5]. The probability for a state of the two-state system

| Decay |                          | CP basis   | Flavor basis   |
|-------|--------------------------|--|--|
| CP=+1 |                          |  |  |
|       | $ K_\zeta\rangle$        | $\frac{1}{\sqrt{1+ \zeta ^2}} \left(  K_1\rangle + \zeta  K_2\rangle \right)$    | $\frac{1}{\sqrt{2(1+ \zeta ^2)}} \left( (1+\zeta)  K^o\rangle + (1-\zeta)  \bar{K}^o\rangle \right)$     |
|       | $ K_{\zeta\perp}\rangle$ | $\frac{1}{\sqrt{1+ \zeta ^2}} \left( -\zeta^*  K_1\rangle +  K_2\rangle \right)$ | $\frac{1}{\sqrt{2(1+ \zeta ^2)}} \left( (1-\zeta^*)  K^o\rangle - (1+\zeta^*)  \bar{K}^o\rangle \right)$ |
| CP=-1 |                          |  |  |
|       | $ K_\zeta\rangle$        | $\frac{1}{\sqrt{1+ \zeta ^2}} \left( \zeta  K_1\rangle +  K_2\rangle \right)$    | $\frac{1}{\sqrt{2(1+ \zeta ^2)}} \left( (1+\zeta)  K^o\rangle - (1-\zeta)  \bar{K}^o\rangle \right)$     |
|       | $ K_{\zeta\perp}\rangle$ | $\frac{1}{\sqrt{1+ \zeta ^2}} \left(  K_1\rangle - \zeta^*  K_2\rangle \right)$  | $\frac{1}{\sqrt{2(1+ \zeta ^2)}} \left( (1-\zeta^*)  K^o\rangle + (1+\zeta^*)  \bar{K}^o\rangle \right)$ |

Table 1: States of the two-state system determined by a decay channel, where the channel has CP plus or minus, and direct CP violation parameter  $\zeta$  (Eq 1). The state is called  $|K_\zeta\rangle$  and is shown in both the CP basis and in the flavor basis. Also shown is the orthogonal state  $|K_{\zeta\perp}\rangle$ . For the  $D^o$  or  $B^o$  cases, replace  $K_1$  by  $D_1$  or  $B_1$ , and  $K^o$  by  $D^o$  or  $B^o$ .

described by a density matrix  $\rho(1)$  to evolve into one given by  $\rho(2)$  is

$$Prob(1 \rightarrow 2) = Tr[\rho(2)\mathcal{M}(t)\rho(1)\mathcal{M}^\dagger(t)], \quad (2)$$

where  $\mathcal{M}$  is the  $2 \times 2$  time evolution operator, given by the exponential of the mass matrix  $M$ ,  $\mathcal{M} = e^{-iMt}$ .  $M$  is in general not hermitian and its eigenstates are not necessarily orthogonal. Eq 2 gives the probability to obtain the state 2 at time  $t$ , having had the state 1 at  $t = 0$ . It applies to the one-body problem of the evolution of a single two-state meson; the application to our two-body problem comes in through the assignment of the states ‘1’ and ‘2’ as in Eq 6.

### 6.3 Density matrices

To use Eq 2 it is necessary to have the density matrix associated with the states. These may be obtained from  $\rho = |K\rangle\langle K|$  using Table 1 and relations like  $|K_1\rangle\langle K_1| = \frac{1}{2}(1 + \sigma_1)$  or  $|K_1\rangle\langle K_2| = \frac{1}{2}(\sigma_3 + i\sigma_2)$ . We use standard identifications of the pauli matrices as in ref[5] where  $\sigma_3$  is the flavor operator:  $\sigma_3 |K^o\rangle = + |K^o\rangle$ ,  $\sigma_3 |\bar{K}^o\rangle = - |\bar{K}^o\rangle$ . In Table 2 we show the density matrices for the states of Table 1.

The density matrices for the orthogonal states “ $\perp$ ” give the ‘initial states’ on the ‘near side’ produced by the ‘collapses’ on the ‘away side’. These are pure states (unless different ‘away sides’ with different  $\zeta$ ’s are averaged together). For  $\zeta = 0$  one sees that these  $\rho$  reduce to the projection operators for the  $CP = \mp 1$  states, namely  $\frac{1}{2}(1 \mp \sigma_1)$ .

Writing the  $\rho$  in the representation

$$\rho(d) = \frac{1}{2}(1 + \mathbf{d} \cdot \boldsymbol{\sigma}) \quad (3)$$

| Channel | Density Matrix $\rho$  | $d_1 = \frac{1}{1+ \zeta ^2} \times$ | $d_2 = \frac{1}{1+ \zeta ^2} \times$ | $d_3 = \frac{1}{1+ \zeta ^2} \times$ |
|---------|--|--------------------------------------|--------------------------------------|--------------------------------------|
| CP=+1   |  |                                      |                                      |                                      |
|         | $\rho(\zeta) =  K_\zeta\rangle \langle K_\zeta $                       | $(1 -  \zeta ^2)$                    | $-i(\zeta - \zeta^*)$                | $(\zeta + \zeta^*)$                  |
|         | $\rho(\zeta_\perp) =  K_{\zeta_\perp}\rangle \langle K_{\zeta_\perp} $ | $-(1 -  \zeta ^2)$                   | $i(\zeta - \zeta^*)$                 | $-(\zeta + \zeta^*)$                 |
| CP=-1   |  |                                      |                                      |                                      |
|         | $\rho(\zeta) =  K_\zeta\rangle \langle K_\zeta $                       | $-(1 -  \zeta ^2)$                   | $i(\zeta - \zeta^*)$                 | $(\zeta + \zeta^*)$                  |
|         | $\rho(\zeta_\perp) =  K_{\zeta_\perp}\rangle \langle K_{\zeta_\perp} $ | $(1 -  \zeta ^2)$                    | $-i(\zeta - \zeta^*)$                | $-(\zeta + \zeta^*)$                 |

Table 2: Density matrices for CP eigenstate decay channels with direct CP violation given by the parameter  $\zeta$  (Eq 1), together with those for the orthogonal state. The  $d$  are to be used in the representation Eq3:  $\rho(d) = \frac{1}{2}(1 + \mathbf{d} \cdot \sigma)$ .

we show the values of  $d$  in Table 2. One has  $\mathbf{d}^2 = 1$  so that  $\rho^2 = \rho$  as needed for a pure-state. One further notes that  $\rho(\zeta)$  and  $\rho(\zeta_\perp)$  are obtained from one another by reversing the ‘spin’ via  $\sigma \rightarrow -\sigma$  or  $\mathbf{d} \rightarrow -\mathbf{d}$ , as should be expected from the ‘spin up-spin down’ analogy. Also the CP=+1 and CP=-1 cases are connected by conjugating with  $\sigma_3$ , that is  $\rho \rightarrow \sigma_3 \rho \sigma_3$ , since  $\sigma_3$  is the CP ‘flip’ operator. Eq3 with  $\mathbf{d}$  a unit vector gives the most general form of a density matrix representing a pure state. This involves two free parameters, corresponding to the real and imaginary parts of  $\zeta$ .

Calling  $\Delta$  the deviation of the  $\mathbf{d}$  from the simple values for CP tags so that  $\mathbf{d} = (d_1, d_2, d_3) = (\pm 1, 0, 0) + \Delta$  we have

$$\Delta = \frac{1}{1 + |\zeta|^2} (\mp 2|\zeta|^2, \pm 2\mathcal{I}\{\zeta\}, 2\mathcal{R}\{\zeta\}), \quad (4)$$

where  $\mathcal{I}\{\zeta\}$  and  $\mathcal{R}\{\zeta\}$  refer to the imaginary and real parts. For the orthogonal states one has  $\mathbf{d} \rightarrow -\mathbf{d}$  and so also a reversal of the sign of  $\Delta$ . The normalization condition  $\mathbf{d}^2 = 1$  implies  $\pm 2\Delta_1 = -\Delta^2$ .

The presence of a  $\Delta_3$  via  $\mathcal{R}\{\zeta\}$  implies a flavor asymmetry which can be induced by the CP violation. This will be manifested below (section 8.2) where we note how a CP tag on the ‘away side’ can lead to a flavor asymmetry at  $t=0$  on the ‘near side’, if  $\mathcal{R}\{\zeta\} \neq 0$ .

## 6.4 Flavor Tag

The  $\zeta$  notation, although perfectly general, is oriented towards decays involving CP eigenstates with a small direct CP violation. However there is also the frequently used lepton tag for flavor eigenstates such as  $K^0$  or  $\bar{K}^0$ , given by decays of the type  $K^0 \rightarrow l^+ \dots$ , where the sign of the lepton implies the sign of the flavor. In this case we will simply use  $\rho = \frac{1}{2}(1 \pm \sigma_3)$ , or  $\mathbf{d} = (0, 0, \pm 1)$  (equivalent to  $\zeta = 1$ , purely real). This is permissible since in the Standard Model flavor change for the flavor eigenstates requires higher order weak interactions, negligible on the order of the effects we discuss here. This assumption has the consequence that with the lepton tag we may reverse the sign of  $\mathbf{d}$  by reversing

the sign of the lepton, a procedure that will prove useful below in constructing various asymmetries. For the CP tag, on the other hand, there is in general no obvious way of accomplishing an experimental reversal of the sign of  $\mathbf{d}$ , once we allow for nonzero  $\Delta$  (see sections 12 or 14).

## 6.5 Completeness

Finally we note the completeness relation

$$I = |\zeta\rangle\langle\zeta| + |\zeta_\perp\rangle\langle\zeta_\perp| = \rho(\zeta) + \rho(\zeta_\perp), \quad (5)$$

which we will use below to exchange a state with its orthogonal state.

## 7 Tags and Rates

Eq2 is the most simple and transparent quantity theoretically. However it is not what can be directly measured experimentally. We suppose an experimental procedure as follows. Let there be a sample where the first, or ‘away side’ decay from the  $l = 1$  boson pair is to a channel ‘a’. The time of each decay establishes a  $t = 0$ . Then in a time interval  $dt$  around a later time  $t$ , we count the number of events in the second or ‘near side’ decay into a channel ‘b’. The number of these second decays is proportional to the time interval  $dt$ , so what we obtain from experiment is a rate quantity we can call  $Rate(b, a; t)$ , for the rate to ‘b’ given ‘a’ at  $t = 0$ . This quantity differs from Eq2 in two ways. First, ‘a’ refers to the ‘away side’, unlike ‘1’ which referred to the “near side”. Secondly, dealing with a rate means we must introduce the rate constant  $\Gamma_b$  which gives the rate of decay into the channel ‘b’ from the eigenstate of the two-state system for ‘b’ decay.

As explained above, ‘a’ on the ‘away side’ implies the orthogonal state  $a_\perp$  on the ‘near side’, whose density matrix is found by reversing the sign of  $\mathbf{d}$ . The formula for the experimental quantity  $Rate(b, a; t)$  is thus

$$Rate(b, a; t) = \Gamma_b Prob(a_\perp \rightarrow b) = \Gamma_b Tr[\rho(\mathbf{d}_b)\mathcal{M}(t)\rho(-\mathbf{d}_a)\mathcal{M}^\dagger(t)], \quad (6)$$

The introduction of the partial rate  $\Gamma$ ’s will in some cases make the examination of simple predictions derived for the  $Prob$  more complicated, but in certain ratios involving different channels the  $\Gamma$ ’s can be made to cancel. A general procedure for finding the  $\Gamma$ ’s experimentally is explained in section 7.1.1.

### 7.1 Time ordering of the ‘Collapse’

An interesting question arises if we consider the two decays— ‘measurements’— separated by a short time, short compared with the internal time evolution as governed by  $\mathcal{M}$ . If a decay ‘b’ takes place shortly after the ‘collapse’ ‘a’ on the other side such that time evolution through the mass matrix has essentially no time to take effect, then we should expect to get a closely related result with



the reverse time ordering. That is, where the ‘b’ ‘collapse’ occurs first. After all, we just have two decays ‘a’ and ‘b’, essentially at the same time. In fact we find that Eq 2 is the same at  $t = 0$ , regardless of which ‘collapse’ is first. We use Eq 5,  $I = \rho(a) + \rho(a_\perp)$  to exchange a state with its orthogonal or  $-\mathbf{d}$  state

$$Tr[\rho(a_\perp)\rho(b)] = Tr[(I - \rho(a))(I - \rho(b_\perp))] = Tr[\rho(b_\perp)\rho(a)], \quad (7)$$

using  $Tr[I] = 2$  and  $Tr[\rho] = 1$ . Thus one obtains the same for Eq 2, regardless of the time ordering, for ‘collapses’ separated by a short time. It is perhaps interesting to note that these arguments are on the level of amplitudes squared and that there is the possibility of a phase factor, presumably unobservable, on the amplitude level. Eq 7 as such, is simply a property of the one-body Eq 2, and is not a priori connected to the two-meson problem. Below we shall further consider the exchange of ‘a’ and ‘b’ when the time interval is not small.

### 7.1.1 Determination of the Partial $\Gamma$

The above point also has an operational interest. While the most simple theoretical quantity is the *Prob* or trace expression Eq 2, the most direct experimental quantity is Eq 6,  $Rate(b, a; t)$ . However, this raises the difficulty that one must know the partial rate  $\Gamma_b$  to obtain the simple *Prob* quantities.

But now Eq 7 gives a method to find the various partial  $\Gamma$ ; or rather their ratios to a given common one. Let the rate be measured for ‘a’ on the ‘away side’, and ‘b’ on the ‘near side’. And inversely let the rate be measured for ‘b’ on the ‘away side’, and ‘a’ on the ‘near side’. The ratio between the two is  $Rate(b, a; t)/Rate(a, b; t)$ . Letting  $t \rightarrow 0$  and using Eq 7

$$\frac{Rate(b, a; t)}{Rate(a, b; t)} \rightarrow \frac{\Gamma_b Tr[\rho(a_\perp)\rho(b)]}{\Gamma_a Tr[\rho(b_\perp)\rho(a)]} = \frac{\Gamma_b}{\Gamma_a} \quad t \rightarrow 0. \quad (8)$$

Thus one may find the ratios of various partial  $\Gamma$ s in terms of directly measurable quantities. One begins with either channel ‘a’ or channel ‘b’ on the ‘away side’, and finds the rate to the other channel on the ‘near side’. Taking the ratio and extrapolating to  $t = 0$ , one finds the ratio of the partial  $\Gamma$ ’s. Doing this for all or many channels, one can obtain all or many of the partial  $\Gamma$  in terms of one of them, a ‘calibration’ rate. We emphasize that the statement on the equality of the traces for  $t \rightarrow 0$  is a consequence of the quantum mechanics alone and does not involve any assumption about symmetry properties of the interactions. We stress that these partial  $\Gamma$ ’s do not have any label for an initial state, since they refer to only one state, the eigenstate for the decay channel.

## 8 Evaluation of the Trace

Given the density matrices and the partial  $\Gamma$ ’s, the rates or *Prob*’s for any process of ‘collapse’ and detection may be evaluated—at least relative to one ‘calibration’ rate.

## 8.1 Expansion of $\mathcal{M}$

It is possible to proceed somewhat further in the evaluation by using a standard identity to expand the  $\mathcal{M} = e^{-iMt}$  matrix. First we remove the scalar part  $\sim I$  of the matrix  $M = m - i\frac{1}{2}\Gamma$ . The real scalar part cancels, leaving the total decay rate  $\Gamma$  so that Eq 2 becomes

$$Prob(1 \rightarrow 2) = e^{-\Gamma t} Tr[\rho(2)e^{-i\mathbf{M}\cdot\sigma t}\rho(1)e^{i\mathbf{M}^*\cdot\sigma t}]. \quad (9)$$

$\mathbf{M}$  is the complex three-vector in the expansion of the traceless part of  $\mathcal{M}$ , namely  $\mathbf{M}\cdot\sigma = (\mathbf{m} - i\frac{1}{2}\Delta\Gamma)\cdot\sigma$ :

$$\mathbf{M} = \mathbf{m} - i\frac{1}{2}\Delta\Gamma. \quad (10)$$

We note  $(\mathbf{M}\cdot\sigma)^\dagger = \mathbf{M}^*\cdot\sigma$ . Now using the identity  $e^{-i\mathbf{b}\cdot\sigma} = \cos b - i(\mathbf{b}\cdot\sigma)\frac{\sin b}{b}$ , one arrives at three terms with different time dependences

$$Prob(1 \rightarrow 2) = e^{-\Gamma t} \left( A | \cos(Mt) |^2 + \left( B \frac{\sin(Mt)}{M} \cos(M^*t) + cc \right) + C \left| \frac{\sin Mt}{M} \right|^2 \right). \quad (11)$$

The ‘cc’ refers to the complex conjugate of the  $B$  term.  $M$  is the complex quantity arising in  $M^2 = \mathbf{M}^2 = (m_1 - \frac{1}{2}i\Delta\Gamma_1)^2 + (m_2 - \frac{1}{2}i\Delta\Gamma_2)^2 + (m_3 - \frac{1}{2}i\Delta\Gamma_3)^2$ . For  $M \approx M^*$ , one may write  $\frac{\sin(Mt)\cos(M^*t)}{M} \approx \frac{\sin(2Mt)}{2M}$ . and  $M$  is one-half the real mass difference. With  $\Delta\Gamma \neq 0$  the trigonometric functions in Eq 11 are to be understood as the full functions of a complex variable. For small  $\Delta\Gamma$ , one has to lowest, linear, order  $M = m - \frac{1}{2}i\frac{\mathbf{m}\cdot\Delta\Gamma}{m}$ , so that  $\mathcal{R}\{M\} \approx m$  and  $\mathcal{I}\{M\} \approx -\frac{1}{2}\frac{\mathbf{m}\cdot\Delta\Gamma}{m}$ . ( $\mathcal{R}$  refers to real part,  $\mathcal{I}$  to imaginary part.) When  $\mathbf{m}$  and  $\Delta\Gamma$  are parallel vectors, as is true for the  $K^0$  system to  $\mathcal{O} \sim 10^{-3}$ , then  $M = m - \frac{1}{2}i\Gamma$ .

For the coefficients one finds

$$\begin{aligned} A &= Tr[\rho(2)\rho(1)] = \frac{1}{2}(1 + \mathbf{d}_2 \cdot \mathbf{d}_1) \\ B &= -iTr[\rho(2)\mathbf{M}\cdot\sigma\rho(1)] \\ &= \frac{1}{2}(\mathbf{d}_1 \times \mathbf{d}_2) \cdot \mathbf{M} - i\frac{1}{2}(\mathbf{d}_1 + \mathbf{d}_2) \cdot \mathbf{M} \\ C &= Tr[\rho(2)\mathbf{M}\cdot\sigma\rho(1)\mathbf{M}^*\cdot\sigma] \\ &= \frac{1}{2}\mathbf{M}\cdot\mathbf{M}^* + \frac{1}{2}(\mathbf{d}_1 - \mathbf{d}_2) \cdot \mathbf{m} \times \Delta\Gamma + \\ &\quad \mathcal{R}\{(\mathbf{d}_2 \cdot \mathbf{M})(\mathbf{d}_1 \cdot \mathbf{M}^*)\} - \frac{1}{2}(\mathbf{d}_2 \cdot \mathbf{d}_1)(\mathbf{M} \cdot \mathbf{M}^*) \end{aligned} \quad (12)$$

With Eq 11 and Eq 12 one has a full, systematic, description of all time dependent phenomena, applying to any of the  $K^0$ ,  $D^0$ , or  $B^0$  systems. These may then be used for  $\phi(1020)$ ,  $\psi(3770)$ , and  $\Upsilon(4s)$  to obtain a  $Rate(b, a; t)$  by inserting

$$\mathbf{d}_1 = -\mathbf{d}_a \quad \mathbf{d}_2 = \mathbf{d}_b, \quad (13)$$

according to Eq 6.

## 8.2 At $t = 0$ , the $A$ term

The ‘near side’ at  $t = 0$  may be viewed as the starting state provided by the ‘preparation of the wavefunction’ resulting from the ‘collapse’ on the ‘away side’, and this is reflected in the  $A$  term of Eq 11, which at  $t = 0$  is the only surviving term.  $A$  is the only coefficient which doesn’t involve  $\mathbf{M}$  and so reflects only the properties of the  $\mathbf{d}$ ’s. In particular with CP tags,  $A$  is sensitive to deviations from the simple  $\mathbf{d} = (\pm 1, 0, 0)$ , showing direct CP violation, as will be discussed for some cases below.

For both ‘away side’ and ‘near side’ channels the same, one has  $\mathbf{d}_2 \cdot \mathbf{d}_1 = -1$  or  $A = 0$ , and so a complete vanishing at  $t=0$ , as expected from the Bose-Einstein statistics.

## 8.3 Manifest Flavor and CP Asymmetries

An interesting configuration for direct CP violation is a CP tag on the ‘away side’ say  $\mathbf{d}_1 = (1, 0, 0) + \Delta$  and the lepton or flavor tag on the ‘near side’,  $\mathbf{d}_2 = (0, 0, \pm 1)$ . One has  $A = \frac{1}{2}(1 \pm \Delta_3)$ . This offers a way to obtain the real part of the direct CP violation parameter  $\zeta$  by taking the difference of the rates for reversed lepton signs. Taking the difference for the two flavor tags one has  $Prob(l^+) - Prob(l^-) = \frac{2}{1+|\zeta|^2} \mathcal{R}\{\zeta\}$ . This may be interpreted as saying that the impure CP state on the ‘away side’ has ‘prepared’, via  $\Delta_3$ , a state on the ‘near side’ which is not completely neutral in flavor. With the lepton tag for flavor and using Eq 6 and assuming that to sufficient accuracy one has  $\Gamma(l^+) = \Gamma(l^-)$  for the leptonic decays, for ‘a’ a  $CP = +1$  tag with parameter  $\zeta$ :

$$\frac{Rate(l^+, a; t) - Rate(l^-, a; t)}{Rate(l^+, a; t) + Rate(l^-, a; t)} = \frac{2}{1 + |\zeta|^2} \mathcal{R}\{\zeta\} \quad t \rightarrow 0. \quad (14)$$

In analogy to this production of a flavor asymmetry produced by a CP tag one may consider a CP asymmetry produced by a flavor tag. With no direct CP violation the opposite side to a flavor tag should have one half of each CP, as reflected by  $A(CP = +1) = A(CP = -1) = \frac{1}{2}$  in Eq 11 following from  $\mathbf{d}_1 \cdot \mathbf{d}_2 = 0$  when there is no direct CP violation. However with nonzero  $\zeta$  we have  $A(CP = +1) = \frac{1}{2} + \Delta_3$ . For the difference of two CP channels  $x$  and  $y$   $A^x - A^y = \Delta_3^x - \Delta_3^y = \frac{2\mathcal{R}\{\zeta^x\}}{1+|\zeta^x|^2} - \frac{2\mathcal{R}\{\zeta^y\}}{1+|\zeta^y|^2}$ . The  $x$  and  $y$  channels may be of same or opposite CP, in any case the nonvanishing of the  $A$  difference is indicative of direct CP violation.

## 9 Exchange of “Collapse and Detection”

Above, in section 7.1, we considered the question of exchanging ‘collapse’ and ‘detection’ for  $t \rightarrow 0$ . We can now examine the question at finite times, with propagation effects included, using the above general expressions Eq 11 and Eq 12. Let there first be a tag with channel ‘a’ and parameter  $\mathbf{d}_a$  on the ‘away side’, and then afterwards a decay into channel ‘b’ on the ‘near side’. Is there

some relation between this and the reverse situation, where the tag is ‘b’ and the later decay is channel ‘a’?

In the exchange of “Collapse” and “Detection” one has the configurations for the density matrix parameters in Eq 12

$$\begin{aligned} \text{‘a’ first : } \quad \mathbf{d}_1 &= -\mathbf{d}_a & \mathbf{d}_2 &= \mathbf{d}_b \\ \text{‘b’ first : } \quad \mathbf{d}_1 &= -\mathbf{d}_b & \mathbf{d}_2 &= \mathbf{d}_a \end{aligned} \quad (15)$$

Under exchange of the two cases,  $B$  in Eq 12 changes sign while  $A$  and  $C$  do not. At the same time, we note that the  $B$  term in Eq 11 is odd under  $t \rightarrow -t$  while the  $A$  and  $C$  terms are even. Hence one can obtain the  $Tr$  expression for ‘a first’ from that for ‘b first’ by putting  $t \rightarrow -t$ .<sup>2</sup>

Thus one may say that the joint process “decay to  $a$  together with decay to  $b$ ” is given by a single function  $f(t)$  via  $e^{-\Gamma t} f(t)$ , with  $t$  or  $-t$  inserted in  $f$  according to whether ‘a’ or ‘b’ is first. (The  $t$  is the time interval between the two decays, and is always positive.) Since  $f(t)$  has both odd and even parts there is no particular relation between  $f(t)$  and  $f(-t)$ , the important point is that  $f$  is smooth, that the odd part vanishes at  $t = 0$ .

Taking the difference of the two configurations only the  $B$  term contributes

$$\begin{aligned} Prob(a_\perp \rightarrow b; t) - Prob(b_\perp \rightarrow a; t) &= 2 \times e^{-\Gamma t} \times (-1) \\ &\quad \frac{1}{2} ((\mathbf{d}_a \times \mathbf{d}_b) \cdot \mathbf{M} - i(\mathbf{d}_a - \mathbf{d}_b) \cdot \mathbf{M}) \frac{\sin(Mt)}{M} \cos(M^* t) + cc. \end{aligned} \quad (16)$$

The expression vanishes identically for  $\mathbf{d}_a \equiv \mathbf{d}_b$  since the two processes then become the same: ‘a’ on the ‘away side’ and ‘a’ on the ‘near side’. By the same token, Eq 16 is therefore sensitive to deviations from the equality of the  $\mathbf{d}$ ’s. This is significant for the detection of direct CP violation since if we consider two channels with the same CP, they have the same  $\mathbf{d}$  unless there is direct CP violation.

## 9.1 Both ‘a’ and ‘b’ are CP channels

With both ‘a’ and ‘b’ channels of the same CP and no direct CP violation one would have  $\mathbf{d}_a = \mathbf{d}_b$  and zero for Eq 16. Eq 16 is thus an experimental quantity representing direct CP violation, independently of CP violation in mixing. Operationally it is obtained by comparing rates for a sample of events with ‘a first’ with a sample for ‘b first’, and then using Eq 6 to find the  $Prob(a_\perp \rightarrow b; t)$  and  $Prob(b_\perp \rightarrow a; t)$ . If the two are not equal (at all times) then there is direct CP violation.

Let there be two channels of the same CP like  $\pi^+\pi^-$  or  $\pi^0\pi^0$  in the  $K^0$  system. Consider the quantity  $\Delta\mathbf{d} = \mathbf{d} - \mathbf{d}' = \mathbf{\Delta} - \mathbf{\Delta}'$  for the difference of their

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<sup>2</sup> The equivalence of the  $\mathbf{d}$  exchanges and  $t \rightarrow -t$  can also be demonstrated formally by the kind of manipulations one uses in connection with the usual  $T$  operation [8]. However, the result here is not a consequence of  $T$  invariance, and holds even if there is a  $T$  violating interaction, such as an  $M_2$  term.

$\mathbf{d}$  parameters. This is only nonzero if there is direct CP violation and is related to the  $\zeta$  as in Eq 4. In Eq 16 it leads to

$$-e^{-\Gamma t} \left( \mathbf{d} \times \Delta \mathbf{d} \cdot \mathbf{M} - i \Delta \mathbf{d} \cdot \mathbf{M} \right) \frac{\sin(Mt)}{M} \cos(M^*t) + cc. \quad (17)$$

Hence by exchanging the order of two channels of the same CP and taking the difference one finds their relative direct CP violation  $\Delta \mathbf{d}$ . With no direct CP violation the result should be zero. In the approximation where direct CP violation or  $\Delta$  is small so  $\mathbf{d}$  is approximately in the ‘1’ direction, one then has that  $\mathbf{d} \times \Delta \mathbf{d}$  is either in the ‘3’ or ‘2’ direction. The ‘3’ component would multiply a CPT violating  $M_3$  and should be very small. This leaves a  $\Delta_3 M_2$  term, which after taking the cc will principally involve  $\mathcal{R}\{\zeta\}$ . Information on  $\mathcal{I}\{\zeta\}$  can come from the second term in Eq 17.

If ‘a’ and ‘b’ are channels of opposite CP and the  $\Delta$ ’s are not large, the second term in Eq 16 will dominate and is proportional to  $M_1$ .

## 9.2 Both ‘a’ and ‘b’ are Flavor Channels, CPT test

In the Standard Model the lepton tag is expected to select a state of definite flavor to high accuracy, so that  $\mathbf{d} = (0, 0, \pm 1)$ . The first term in Eq 16 vanishes and the second term is given by the CPT forbidden  $M_3$ . Hence the non-vanishing of

$$Rate(l^-, l^+; t) - Rate(l^+, l^-; t) \sim i M_3 \frac{\sin(Mt)}{M} \cos(M^*t) + cc, \quad (18)$$

indicates a CPT violation, essentially proportional to  $\mathcal{I}\{M_3\}$ . One has assumed  $\Gamma_{l^+} = \Gamma_{l^-}$  and  $\mathbf{d} = (0, 0, \pm 1)$  for the flavor tags. Since this test amounts to the difference between a flavor and an anti-flavor process, it corresponds to a known test in  $K^0$  physics [13].

## 9.3 One Flavor, One CP Channel, Identification of $\mathcal{I}\{\zeta\}$

In this case Eq 16 can be non-zero with no direct CP violation. The first term is proportional to the  $T$  violating  $M_2$  and the second to the  $T$  conserving  $M_1$ . This first term is what arises in simple CP and  $T$  tests in the  $B^0$  system [5].

Deviations from these simple values are of interest in obtaining  $\Delta_2$ , or  $\mathcal{I}\{\zeta\}$ . In Eq 16 the  $(\mathbf{d}_a \times \mathbf{d}_b)$  term, as said, is in the ‘2’ direction without direct CP violation and so is proportional to  $M_2$ , as in the simple CP,  $T$  tests discussed in ref[5]. However with direct CP violation  $(\mathbf{d}_a \times \mathbf{d}_b)$  will contain a term  $\pm \Delta_2$  in the ‘1’ direction, proportional to  $M_1$ . The resulting deviation from the simple, purely mixing induced, predictions for these tests thus provides a measurement of  $\mathcal{I}\{\zeta\}$ .

# 10 Same ‘away side’ and ‘near side’

In the previous section 9 we considered the exchange of ‘collapse’ and ‘detection’ channels. Continuing with some general features, we note that a particularly

simple situation arises when both are the same channel, the ‘same on both sides’ configuration. Then we have  $\mathbf{d}_1 = -\mathbf{d}_2$  and so Eq 12 becomes

$$\begin{aligned} A &= 0 \\ B &= 0 \\ C &= \mathbf{M} \cdot \mathbf{M}^* - \mathbf{d} \cdot \mathbf{m} \times \Delta \mathbf{\Gamma} - (\mathbf{d} \cdot \mathbf{M})(\mathbf{d} \cdot \mathbf{M}^*) \end{aligned} \quad , \quad (19)$$

where  $\mathbf{d} = \mathbf{d}_1$  gives the density matrix for the ‘away side’ or equivalently the final state on the ‘near side’.

The vanishing of A and B in Eq 19 is the generalization of the statement that the same state cannot appear on both sides at  $t=0$ . In particular this leads to the feature that the rate is given by C alone, so that the small  $t$  behavior for the rate is necessarily  $\sim t^2$  for any choice of channel.

These formulas are completely general, and in particular independent of the presence of direct CP violation with a nonzero  $\zeta$  or not. Thus *any* data set with the same ‘away side’ and ‘near side’ will have the same  $e^{-\Gamma t} \left| \frac{\sin(Mt)}{M} \right|^2$  behavior. This parallel behavior applies to all three systems  $(\phi, \psi, \Upsilon)$  and to all channels, independent of CP or CPT questions, and provides a test of the quantum mechanical procedure.

Since  $M$  and  $\Gamma$  are general properties of the system, the only difference between various channels used will be in their  $C$  parameters and of course their partial decay rates, which can be found via the method of section 7.1.1. If we compare two channels  $x$  and  $y$ , both used in the ‘same on both sides’ configuration, one has

$$\frac{\text{Rate}(x, x; t)}{\text{Rate}(y, y; t)} = \frac{\Gamma_x C(x)}{\Gamma_y C(y)}, \quad (20)$$

constant in time.

## 10.1 Lepton tag

Continuing with the same ‘near side’ and ‘away side’ configurations, one expects to high accuracy that a lepton tag as in  $K^0 \rightarrow l^\pm + \dots$  specifies the flavor, and so  $\mathbf{d}$  to be  $(0, 0, \pm 1)$ . Therefore one has from Eq 19 that  $C$  becomes

$$C(l^\pm) = \mathbf{M} \cdot \mathbf{M}^* - \pm (\mathbf{m} \times \Delta \mathbf{\Gamma})_3 - |M_3|^2 = |M_1|^2 + |M_2|^2 - \pm (\mathbf{m} \times \Delta \mathbf{\Gamma})_3 \quad (21)$$

where a possible CPT violating  $M_3$  term cancels out. Thus in the difference between lepton signs only the cross term survives and one will have

$$\begin{aligned} \frac{\text{Rate}(l^+, l^+; t) - \text{Rate}(l^-, l^-; t)}{\text{Rate}(l^+, l^+; t) + \text{Rate}(l^-, l^-; t)} &= \frac{\Gamma_{l^+} C(l^+) - \Gamma_{l^-} C(l^-)}{\Gamma_{l^+} C(l^+) + \Gamma_{l^-} C(l^-)} \\ &= \frac{C(l^+) - C(l^-)}{C(l^+) + C(l^-)} = \frac{(\mathbf{m} \times \Delta \mathbf{\Gamma})_3}{|M_1|^2 + |M_2|^2} \quad , \quad (22) \end{aligned}$$

where we use  $\Gamma_{l+} = \Gamma_{l-}$ . Thus in the ‘same on both sides’ configurations this kind of ratio can be used to obtain information on  $\Delta\Gamma$ . Or more precisely on to the extent that  $\Delta\Gamma$  and  $\mathbf{m}$  are not parallel, i.e do not commute. For the  $B^o$ , Eq22 should be very small, in  $\psi(3770) \rightarrow D^o D^o$  it may help to obtain information on the  $D^o$  mass matrix.

A point to note here is that the numerator in Eq22 is bounded. Since  $(\mathbf{m} \times \Delta\Gamma)_3 \leq m\Delta\Gamma$ , there is a limit to the ratio, which presumably can be determined in a complete experimental analysis. Also  $(\mathbf{m} \times \Delta\Gamma)$  has only the ‘3’ component when CPT is good, since the both vectors lie in the ‘1-2’ plane.

In the presence of direct CP violation, the analogous statement to Eq22 is not rigorously possible with CP tags. This is because if a given decay channel corresponds to a certain  $\mathbf{d}$ , there is not necessarily another channel that corresponds to  $-\mathbf{d}$ . This is further discussed in section 14.

## 10.2 $\Delta\Gamma \approx 0$

When we can put  $\Delta\Gamma \approx 0$ , as for the  $B^o$  system, C in Eq19 further simplifies:

$$C = m^2 - (\mathbf{d} \cdot \mathbf{m})^2 \quad \Delta\Gamma \approx 0 \quad (23)$$

For CP tags with no direct CP violation,  $\mathbf{d}$  is  $(\pm 1, 0, 0)$  and so  $C$  is constant from channel to channel. A variation is then indicative of direct CP violation. For direct CP violation negligible, and assuming good CPT one will thus have  $C = m_2^2$ , the square of the CP, T violating mass term. This affords a direct measurement of  $m_2$  to the accuracy allowed by the assumptions  $\Delta\Gamma \approx 0$  and  $\Delta \approx 0$ . Deviations from this simple value at more than the  $10^{-3}$  level [9] should give information on the other, direct CP violation components of the  $\mathbf{d}$ . This should be one of the simplest ways to observe direct CP violation in the  $B^o$  system.

## 10.3 Same CP on Both Sides

A variation on the ‘same on both sides’ configuration is ‘same CP on both sides’. If one takes channels of the same CP, like  $\pi^+\pi^-$  and  $\pi^o\pi^o$  for the  $K^o$ , in the absence of direct CP violation they both define the same state of the  $K^o$ . Then one has the ‘same on both sides’ configurations, as discussed in the earlier parts of this section. In particular one will have  $A = 0$  and the corresponding vanishing of the rate towards  $t = 0$ . On the other hand, with direct CP violation the two are no longer identical systems, and the rate need not vanish towards  $t = 0$ . In particular one has, with parameters  $\mathbf{d}$  and  $\mathbf{d}'$ ,

$$A = \frac{1}{2}(1 - \mathbf{d} \cdot \mathbf{d}') = (1/4)(\Delta - \Delta')^2 \quad (24)$$

where we used the normalization relation mentioned after Eq4.

This gives, via Eq11, a non-vanishing rate at  $t=0$ . This was essentially Lipkin’s proposal [2]. The value at  $t = 0$  measures the “non-identity” of the two channels. It is amusing that is possible, by means of this process, to give a quantitative measure of the extent to which two different states are ‘identical’.

## 11 $B^o$ System

Another general simplification ensues when one may take  $\Delta\Gamma \approx 0$ . In the  $\Upsilon(4s) \rightarrow B^o B^o$  system this is believed to be true to about the  $10^{-3}$  level [9]. Setting  $\Delta\Gamma = 0$  one has  $\mathbf{M}$  real,  $\mathbf{M} = \mathbf{m}$  and  $\frac{1}{2}\Delta M = \frac{1}{2}\sqrt{m^2}$  real numbers and so the ordinary trigonometric functions in Eq 11. In particular one now has for the  $B$  term  $(B + B^*)\cos(mt)\sin(mt)/m = (B + B^*)\sin(2mt)/2m$  and the coefficients

$$\begin{aligned} A &= \frac{1}{2}(1 + \mathbf{d}_1 \cdot \mathbf{d}_2) \\ B + B^* &= (\mathbf{d}_1 \times \mathbf{d}_2) \cdot \mathbf{m} \\ C &= \frac{1}{2}m^2(1 - \mathbf{d}_1 \cdot \mathbf{d}_2) + (\mathbf{d}_1 \cdot \mathbf{m})(\mathbf{d}_2 \cdot \mathbf{m}) \end{aligned} \quad (25)$$

The general configurations examined above may be discussed here with these simplified forms. The same ‘away side’ and ‘near side’ was discussed in section 10.2.

The exchange of ‘near side’ and ‘away side’ of section 9 now becomes

$$\begin{aligned} Prob(a_\perp \rightarrow b; t) - Prob(b_\perp \rightarrow a; t) &= 2 \times e^{-\Gamma t} \times (-1) \\ &\quad (\mathbf{d}_a \times \mathbf{d}_b) \cdot \mathbf{m} \frac{\sin(2mt)}{2m}. \end{aligned} \quad (26)$$

With two flavor tags,  $\mathbf{d} = (0, 0, \pm 1)$ , this quantity is zero. If it were nonzero at more than the  $10^{-3}$  level, this could indicate a larger than expected value for  $\Delta\Gamma$ .

With two CP tags,  $\mathbf{d} = (\pm 1, 0, 0) + \mathbf{\Delta}$ , the leading terms would be linear in the  $\mathbf{\Delta}$ . The ‘2’ component of  $\mathbf{\Delta}_a - \mathbf{\Delta}_b$  multiplies the CPT violating  $M_3$ , so that largest term should be  $(\Delta_a - \Delta_b)_3 M_2$ . Since the CP, or T violating  $M_2$  is not small [5], this offers a way to obtain the channel differences  $\mathcal{R}(\zeta_a - \zeta_b)$ . There is also a  $(\Delta_a \times \Delta_b)$  term, presumably small.

The case of one flavor tag and one CP tag will be discussed in section 12.

## 12 $T$ tests

A simple and intuitive  $T$  test consists in comparing a certain process “forwards” and “backwards”. In ref [5], where direct CP violation was neglected, in section 5, “T asymmetry”, for example, one compared  $B^o \rightarrow B_2$  with  $B_2 \rightarrow B^o$ . In the language we use here, both particles were on the ‘near side’. With neglect of direct CP violation one could hope to identify the  $B_2$  in the final state by a channel thought to be predominately CP=-1 and to produce it in the initial state by a channel thought to be predominately CP=+1 on the ‘away side’.

However with direct CP violation taken into account, as we do here, it is not clear how to do this. Given a certain final channel on the ‘near side’, there is no certain means to make this also the initial state on the ‘near side’ by some ‘away side’ tag.



However we can propose an analogous test here, with direct CP violation taken fully into account. One uses the exchange of ‘near side’ and ‘away side’ as examined in section 9. The quantity Eq 16 resembles that used in the discussion of the previous  $T$  tests. However the  $\mathbf{d}$  now include a possible direct CP violation via  $\Delta$ , Eq 4.

Applying Eq 16 to the  $\Upsilon(4s) \rightarrow B^0 B^0$  case, with neglect of  $\Delta\Gamma$ , one has that the effect is simply given by  $(\mathbf{d}_a \times \mathbf{d}_b) \cdot \mathbf{M}$ , Eq 26. With two flavor tags the  $\mathbf{d}$  are both in the  $\pm 3$  direction and one obtains zero [10]. We thus consider the flavor tag-CP tag configuration, In the limit of no direct CP violation for this configuration one has  $\mathbf{d}_a \times \mathbf{d}_b$  in the ‘2’ direction, and the effect is proportional to  $m_2$ , the CP, T violating term in the mass matrix. One thus obtains essentially the same result as in ref [5]. Here, including direct CP violation with a nonzero  $\Delta$  for the CP tag, there is also a term  $\Delta_2 m_1$ .

Thus in the  $\Upsilon(4s) \rightarrow B^0 B^0$  system an asymmetry between the cases ‘flavor tag first, CP tag second’ and vice-versa requires either an  $m_2$  or an imaginary direct CP violation, to the level that  $\Delta\Gamma$  can be neglected. A detailed measurement of the effect would be of interest since finding the  $\Delta_2 m_1$  contribution would provide a way of obtaining  $\mathcal{I}\{\zeta\}$ , while most of our simple effects are proportional to  $\mathcal{R}\{\zeta\}$ .

## 13 Two-Channel Differences

As mentioned earlier, one of the simplest manifestations of direct CP violation is in the comparison of two ‘away side’ channels of the same CP. *Any* difference at all on the ‘near side’ indicates direct CP violation.

In our formulas, these differences result from having different  $\Delta$ ’s for the  $\mathbf{d}$  of the two ‘away side’ tags. Since the final state on the ‘near side’ is the same in the two cases,  $\Gamma_b$  will drop out in a ratio such as

$$\frac{\text{Rate}(b, a; t) - \text{Rate}(b, a'; t)}{\text{Rate}(b, a; t) + \text{Rate}(b, a'; t)} = \frac{\text{Prob}(b, a; t) - \text{Prob}(b, a'; t)}{\text{Prob}(b, a; t) + \text{Prob}(b, a'; t)}, \quad (27)$$

where we call the different ‘away side’ channels of the same CP  $a$  and  $a'$ .

Any nonzero value of the experimental quantity on the lhs indicates direct CP violation. The connection to  $\mathbf{d}_a$  and  $\mathbf{d}_{a'}$  and thus the  $\zeta$  may be found by writing  $\mathbf{d}_a = (0, 0, \pm 1) + \Delta_a$  and  $\mathbf{d}_{a'} = (0, 0, \pm 1) + \Delta_{a'}$ , and finding the differences in the coefficients A,B,C when (the negative of) these are inserted in Eq 12 for  $\mathbf{d}_1$  and one sets  $\mathbf{d}_2 = \mathbf{d}_b$ .

## 14 CPT Tests with CP tags

Good CPT requires that the ‘3’ component of  $\mathbf{M}$  be zero,  $M_3 = 0$ , which also implies that  $\mathbf{m} \times \Delta\Gamma$  has no ‘1,2’ components. In searching for such a forbidden component, in section 9.2 we used the property that with a lepton tag it is possible to reverse the sign of  $\mathbf{d}$  by reversing the sign of the lepton.

With CP tags this would be also possible to the extent that one neglects direct CP violation for the decay channels, that is one puts  $\Delta = \mathbf{0}$ . Then one may reverse  $\mathbf{d}$  by simply choosing a channel of the opposite CP. For example in Eq 19, one has the term  $\mathbf{d} \cdot \mathbf{m} \times \Delta \Gamma$ , which is odd under a reversal of  $\mathbf{d}$ . Let the *Prob*'s and so the *C*'s in the 'same on both sides' configurations be determined for two channels of opposite CP, which we designate with '+' and '-'. Then

$$\frac{Prob(+)-Prob(-)}{Prob(+)+Prob(-)} = \frac{C(+)-C(-)}{C(+)+C(-)} = \frac{(\mathbf{m} \times \Delta \Gamma)_1}{|M_2|^2 + |M_3|^2} \quad \Delta \approx 0 \quad (28)$$

provides an expression sensitive to the CPT forbidden  $(\mathbf{m} \times \Delta \Gamma)_1$ .

This test is only good only to the level that the assumption of no direct CP violation, that is  $\Delta = 0$ , holds. However it may be possible with extensive data and analysis of the type described in this paper that the values of  $\Delta$  can be established in some channels to good accuracy. Then inserting these in Eq 19, the necessary corrections to Eq 28 could be found.

Similar arguments apply to other instances where the forbidden  $M_3$  or  $(\mathbf{m} \times \Delta \Gamma)_{1,2}$  appear. In fact, it is conceivable that the appearance of nontrivial  $\Delta$  makes new kinds of tests possible. For instance in the above example, if a large  $\Delta_2$  is established for some channel, then there would be a sensitivity to the CPT violating  $(\mathbf{m} \times \Delta \Gamma)_2$ .

## 15 The $l = 0$ System

The  $l = 0$  system, where the internal wavefunction will be symmetric, instead of antisymmetric, is of conceptual interest, although it is not clear if there is a useful experimental situation of this type. It would result from the decay of a spin-zero resonance into a pair of  $K^0$   $D^0$  or  $B^0$  particles. We consider it briefly as an illustration of the general scope of the problem.

The  $l = 1$  case with the antisymmetric internal state, as we have discussed at great length, is actually the simpler case. For the following reason. There is only one antisymmetric state of two two-state systems. In the  $K^0$  notation this is  $\frac{1}{\sqrt{2}}(|K\rangle |\bar{K}\rangle - |\bar{K}\rangle |K\rangle)$ . In the spin-1/2 analogy this is the singlet state, with total 'angular momentum' zero, of the two 'spins'. Any evolution will simply return the same state since the evolution – rotation of the 'spins' – will respect the symmetry and there is just this one state. On the same grounds, this state expanded in terms of the state for a given tag  $|K_\zeta\rangle$ , always has the same form, namely  $\frac{1}{\sqrt{2}}(|K_\zeta\rangle |K_{\zeta\perp}\rangle - |K_{\zeta\perp}\rangle |K_\zeta\rangle)$ . This observation is the basis of our repeated use of the fact that the state recoiling against a given tag is always the state orthogonal to that tag.

With a symmetric internal state, the situation is different. There are three possible symmetric states, in the angular momentum analogy  $S=1$  with  $S_3 = \pm 1, 0$ . Namely  $(|K\rangle |K\rangle, |\bar{K}\rangle |\bar{K}\rangle, \text{ and } \frac{1}{\sqrt{2}}(|K\rangle |\bar{K}\rangle + |\bar{K}\rangle |K\rangle)$ . True, due to flavor conservation in strong interactions the pair will be 'born' in this last, flavor neutral, state. In the spin analogy this is an eigenstate of  $S_3$  with eigenvalue

zero. Indeed, in the limit of no CP violation the effect of the mass matrix is to induce a rotation around the ‘1’ axis and this state is invariant—so that in this limit one would always retain the same state. However, with CP violation the rotation is not just around the ‘1’ axis and other components will develop with time. Furthermore, the form of an expansion in terms of  $|K_\zeta\rangle$  states will not be invariant, as it is in the antisymmetric case. Hence in general the state recoiling against a given tag will be some time-dependent combination of the same state and its orthogonal state. And not simply the same state, as one might have naively supposed.

A complete analysis of this problem would involve starting from a  $t = 0$  defined by the decay of the parent resonance and determining the subsequent evolution of the two-body system due to the mass matrices. While for the  $l = 1$  cases it is sufficient to count the time from the decay of one member of the pair, here the analysis would have to start with the decay of the parent resonance, and it is not clear if this time can be determined sufficiently well experimentally. If a suitable resonance is found this problem might be worth further discussion.

This example shows that in general the identification of a recoiling state in terms of an ‘away side’ tag is not always elementary and must be examined on a case-by-case basis.

## 16 Fundamentals of quantum mechanics

Our arguments are within the usual canon of quantum mechanics, except, perhaps, that here the “measurement” is via a spontaneous decay process and does not involve any obvious active disturbance by an ‘observer’. Nevertheless, the arguments use the “collapse” —or its equivalent in terms of amplitudes [6])— in very fine, perhaps unprecedented, detail. The confirmation and consistency of our predictions, here as well as those in ref[5], would provide an impressive verification of quantum mechanical principles.

### 16.1 Test of Consistency of the ‘Collapse’ Treatment

Our most important assumption is that a decay fixes the ‘away side’ of the  $K^0$   $D^0$  or  $B^0$  pair. Out of the continuous range of possibilities available for the state vector of the two-state system, the ‘away side’ state is fixed to be one only, the eigenstate for the decay channel. It is here that there appears to be an abrupt change from ‘potentialities’ to ‘certainty’.

While it is difficult to imagine another way of doing this, one might entertain the thought that the ‘away side’ decay does not imply the corresponding eigenstate (see section 4) with exactly probability 1; perhaps the factor could be channel dependent or dependent on the ‘near side’ state.

It is thus interesting that there is a test of the consistency of this assumption. This arises from the observation that there is more than one way to arrive at the ratio of two partial rates. As discussed in sect.7.1.1, one can find  $\Gamma_a/\Gamma_b$  by taking the ratio of rates for ‘a’ on the ‘away side’ and ‘b’ on the ‘near side’ to

that for the inverted situation, and letting  $t \rightarrow 0$ . Let us call the ratio of partial rates determined by this experimental procedure  $\left\{\frac{\Gamma_a}{\Gamma_b}\right\}$ :

$$\left\{\frac{\Gamma_a}{\Gamma_b}\right\} \equiv \frac{Rate(b, a; t)}{Rate(a, b; t)} \Big|_{t \rightarrow 0} \quad (29)$$

Now it should be possible to find the same ratio in a roundabout way via another pair of processes, as in

$$\left\{\frac{\Gamma_a}{\Gamma_b}\right\} = \left\{\frac{\Gamma_a}{\Gamma_c}\right\} \times \left\{\frac{\Gamma_c}{\Gamma_b}\right\} \quad (30)$$

The procedures implied on the right are different from that on the left and involve different channels, and it is perhaps conceivable that experiment leads to different numbers for the two sides of Eq 30. Thus Eq 30 presents an experimental test of our method and in particular of our treatment of the “collapse of the wavefunction”. Naturally if tests of the type Eq 30 are experimentally consistent it does not necessarily imply the veracity of the method. But a clear breakdown of Eq 30, or its generalizations, would be very interesting and necessitate a great rethinking of the problem.

Finally, concerning the ultimate meaning of the “collapse of the wavefunction”, we would like to take this opportunity to reiterate our view [6] that the “collapse of the wavefunction” is a convenient fiction that arises due to an unnecessary reification of the wavefunction. The need for it goes away in an amplitude approach to quantum mechanics, where there is nothing to ‘collapse’ in the first place. However, as one sees in the present application, it is a very convenient fiction, often allowing a quick and easy insight in seemingly complicated situations.

## 17 Conclusions

We have pursued the idea that the observation of a decay of a two- state system like  $K^0 D^0$  or  $B^0$  amounts to a “measurement” that fixes its internal state. In the  $l = 1$  decays  $\phi(1020) \rightarrow K^0 K^0$ ,  $\psi(3770) \rightarrow D^0 D^0$ , or  $\Upsilon(4s) \rightarrow B^0 B^0$ , Bose-Einstein statistics then determines the recoil at the time of the decay to be the orthogonal state. This implies that the recoiling state will contain information on direct CP violation in the first or ‘away side’ decay. A general parameterization for such effects is given and then applied to the further evolution of the recoiling state, called the ‘near side’. The parameterization gives a clear separation of direct CP violation effects and mixing-induced CP violation.

The result of the analysis is a rich phenomenology and some configurations of special interest are identified, particularly for studying direct CP violation. These include exchange of ‘near side’ and ‘away side’, ‘same on both sides’, same CP on both sides, and comparison of two ‘away side’ tags of the same CP.

Examination of a hypothetical analogous case with  $l = 0$  shows that in general the identification of the state of a recoil by this method is not always elementary.

The method involves a quite detailed use of the “collapse of the wavefunction”, and experimental results on the many predictions would provide tests of the underlying ideas. A consistency test for the treatment of the “collapse of the wavefunction”, which can be carried out by a certain determination of partial decay rates, is suggested.

## 18 Appendix

Traditional notation for direct CP violation has been rather heterogeneous and differs from case to case, often combining the effects of direct CP violation and mixing CP violation in one parameter. Our notation, using the  $\zeta$  parameter as expressed through  $\Delta$ , Eq 4, offers a systematic, uniform description, which applies to all cases. With  $\Delta$  or equivalently  $\zeta$  zero, there is no direct CP violation. Mixing-induced CP violation is given by  $M_2$  the coefficient of  $\sigma_2$  in the mass matrix,  $\mathbf{M} \cdot \sigma = (\mathbf{m} - i\frac{1}{2}\Delta\mathbf{\Gamma}) \cdot \sigma$ : The essential difference in the notations is that ours, adapted to the “collapse” viewpoint, refers to the CP eigenstates, while the conventional notation refers to the quasi-stationary states such as  $K_S, K_L$ . Since the quasi-stationary states are defined by the mass matrix, there is inevitably a combination of effects when these states are used.

Thus while  $\zeta$  resembles the conventional  $\eta$  in its definition, see [11],  $\eta$  contains effects both due to mixing in the mass matrix (the  $\epsilon$  parameter) and direct CP violation (the  $\epsilon'$  parameter). The connection between an  $\eta$  and a  $\zeta$  can be obtained by expanding the states like  $K_S, K_L$  in terms of the  $p, q$  parameters. This leads to

$$\eta = \frac{\frac{1}{\sqrt{2}}(p - q) + \frac{1}{\sqrt{2}}(p + q)\zeta^*}{\frac{1}{\sqrt{2}}(p + q) + \frac{1}{\sqrt{2}}(p - q)\zeta^*}. \quad (31)$$

for a CP=+1 channel. For a CP=-1 channel, the sign of  $q$  is reversed.

One notes that in the no-CP violation -in-mixing limit, where  $p = q$ , one has  $\eta = \zeta^*$ , as should be expected in accordance with the role of  $\zeta$  as purely characterizing direct CP violation. Otherwise  $\eta$  combines both mixing and direct CP violation effects.

In the limit of small  $(p - q)$  and small  $\zeta$ , as in the  $K^0$  system, one has, for example for the  $\pi^0\pi^0$  or the  $\pi^+\pi^-$  channels

$$\eta_{\pi^0\pi^0} = \frac{p - q}{p + q} + \zeta_{\pi^0\pi^0}^* \quad \eta_{\pi^+\pi^-} = \frac{p - q}{p + q} + \zeta_{\pi^+\pi^-}^* \quad (32)$$

With the traditional notation where  $\eta_{\pi^0\pi^0} = \epsilon - 2\epsilon'$  and  $\eta_{\pi^+\pi^-} = \epsilon + \epsilon'$ , one has  $\epsilon' = \frac{1}{3}(\eta_{\pi^+\pi^-} - \eta_{\pi^0\pi^0}) = \frac{1}{3}(\zeta_{\pi^+\pi^-}^* - \zeta_{\pi^0\pi^0}^*)$ . This shows that a nonzero  $\epsilon'$  requires not only direct CP violation but also a difference between the two channels involved. With strong CP violation in the mass matrix, as in the  $B^0$  system, the full relation Eq 31 must be used to connect  $\zeta$  and  $\eta$ .

For another example, now in the  $B^0$  system, ref [3], in Eq.10.38, following [12], discuss a flavor tag on the ‘away side’, followed by detection of a CP state on the ‘near side’. The rate at  $t = 0$  (their  $\Delta t = 0$ ) is given by an  $|A(f)|^2$  where

‘f’ refers to the final CP tag. In our notation this is given by the  $A$  coefficient in Eq 25,  $A = \frac{1}{2}(1 + \Delta_3) = \frac{1}{2} + \mathcal{R}\{\zeta\}$ . Thus if the final tag involves a direct CP violation it has been implicitly incorporated in  $A(f)$ , while in our notation it is explicitly exhibited. As explained in section 8.2, this direct CP violation, proportional to  $\mathcal{R}\{\zeta\}$  can be observed by reversing the sign of the lepton tag or by comparing two different tags of the same CP.

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